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## STRESS-INDUCED ANISOTROPY

stants. If shape anisotropy is ignored, uniaxial anisotropy will be obtained only if either  $\lambda_{100}/\lambda_{111} > 0$  or  $\lambda_{100}/\lambda_{111} < 0$  with  $|\lambda_{111}| > |\lambda_{100}|$ . With compressive stress, both cases would require  $\lambda_{111} > 0$  in addition to the above.

For  $\sigma$  in the (111) plane, the sum of Eqs. (3) and (17) gives

$$E^{111} = K_1(\sin^4\theta \sin^2\phi \cos^2\phi + \sin^2\theta \cos^2\theta) + \sigma\lambda_{100}$$

-  $(\sigma \lambda_{111} - \frac{4}{3}\pi M^2) [\sin^2 \theta \sin \phi \cos \phi + \sin \theta \cos \theta (\sin \phi + \cos \phi)]$ 

$$+\frac{2}{3}\pi M^2$$
 (25)

By setting  $\partial E^{111}/\partial \theta = \partial E^{111}/\partial \phi = 0$ , it may be shown that the pertinent extrema move in the three {110} planes which contain the [111] axis. Since these three situations are equivalent, the problem may be solved for only one of them (as with  $\sigma$  in the (001) plane) by setting  $\phi = \frac{\pi}{4}$ . The first result is that the extremum along the [111] axis does not move. However, the movements of the other extrema in the (110) plane (i.e.,  $\phi = \frac{\pi}{4}$ ) are given by

$$(\sigma\lambda_{111} - \frac{4}{3}\pi M^2)/K_1 = \frac{\sin\theta\cos\theta(3\cos^2\theta - 1)}{\sin\theta\cos\theta + \sqrt{2}(\cos^2\theta - \sin^2\theta)}$$
(26)

which is plotted in Fig. 3 as  $\theta$  is varied from zero to  $\pi$ . As the ordinate increases in a positive sense, the extremum at [001] rotates towards the normal [111] direction until it meets the extremum from the [110] axis and both disappear for  $(\sigma\lambda_{111} - \frac{4}{3}\pi M^2)/K_1 \ge 0.51$ . At the same time, the [111] extremum rotates into the plane at  $\theta \simeq 145^\circ$ , which is a pole in Eq. (26), thus requiring that  $(\sigma\lambda_{111} - \frac{4}{3}\pi M^2)/K_1 \Rightarrow \infty$  in order to reach the plane. Where  $(\sigma\lambda_{111} - \frac{4}{3}\pi M^2)/K_1$  increases in a negative direction, the [001] extremum rotates toward the pole at  $\theta \simeq 145^\circ$ , and the [110] and [111] extrema merge and vanish at  $\theta \simeq 110^\circ$  for  $(\sigma\lambda_{111} - \frac{4}{3}\pi M^2)/K_1 \le -0.15$ . These conditions are depicted in Fig. 4.

For convenience, all of the conditions for uniaxial anisotropy discussed above are listed in Table 1. In Table 2, the expressions for E along the normal and different directions in the plane are given together with the differences in energy which represent the effective uniaxial anisotropy constants  $K_u$ . These results may be used to calculate  $K_u$  once the uniaxial anisotropy conditions have been satisfied.

## Discussion and Conclusions

From the results summarized in Table 1, it is evident that the effect of



Variation of  $(\sigma\lambda_{111} - \frac{4}{3}\pi M^2)/K_1$  with the polar angle  $\theta$  in the (110) plane for compressive stress  $\sigma$  in the (111) plane. This curve is a plot of Eq. (26) for  $0 < \theta < \pi$ .



Movements of important energy extrema and conditions for uniaxial anisotropy with stress in the (111) plane, for (a)  $K_1 < 0$  and (b)  $K_1 > 0$ . Activity in only one of the three pertinent {110} planes is shown and shape anisotropy is assumed to be neglibible.